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Heliocentrism vs. Geocentrism:

The battle between the new and the old

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Absract:

This essay is about the conflict of two ideas, a new and an old.

The Greek astronomers conducted good observations of the celestial bodies. The Greeks believed in a geocentric universe and a geometrically perfect world. Their world view was carried on long after their golden age and there theories stood for two thousands years. Still there were some Greek astronomers who anticipated the events which took place two thousand years later. Aristarchus of Samos proposed a heliocentric model somewhere around 300 BC and anticipated Copernicus.

The research question is: Could Aristotle and Aristarchus describe the motion of the celestial bodies and the earth in a geocentric and heliocentric universe respectively? I will compare the two models in the essay, analyse and answer the question

In the geocentric model, gravity is explained. The earth is at the centre since it is the heaviest, and the motions of the planets are described in a complex way using circles.
In the heliocentric model, the earth is orbiting the sun because it is much larger and it rotates about its own axis.
The lack of inertia leads to the arrow argument, saying that an arrow which is shot straight up lands on the same spot. This would not occur if the earth was spinning.
This and the lack of stellar parallax break down the model.

It is definitely possible to reach a heliocentric conclusion but the problems that arise are huge and it is hard to explain. It is not until the development of physics, the understanding of inertia that heliocentrism wins.

words: 259

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Introduction:

In physics there is always a struggle between new and old theories. Sometimes we come to a crossroad; it becomes necessary to either stick with the old or change to accept the new. Einstein’s special theory of relativity is such a case. Physics had reached the limit of mechanics and the theory of relativity was necessary to conform with new discoveries. At first his theories were not widely accepted, due to their extreme controversy but today his theories are not only accepted but also highly praised. More than two thousand years ago there was another conflict: The heliocentric versus the geocentric model of the universe.

The source of modern physics is ancient Greece. Therefore I thought it was an interesting topic to study in my extended essay. At first I thought that Greek science was naïve and ignorant. That they thought the earth was flat and in the centre of the universe. So my first approach was to show how you can easily prove that the earth is spherical. When I started researching it turned out that the Greek knew that the earth was spherical and many of their astronomical calculations were extremely good. They had calculated the moon orbit wrong to less than one second[1] compared to today’s values. I realised that their limitation did not lay in their observations. I thought it would be interesting to study why they did not manage to develop their physics and in particular, something they are often blamed for: why did they insist in the geocentric-model?

There are many Greek scientist and philosophers. I will focus on two, Aristotle and Aristarchus. Aristotle reasoned on the subject and came up with claims and counter claims. Aristarchus, a later Greek, is the first to seriously present the heliocentric model of the universe. I will also study two of his contemporise, Archimedes and Eratosthenes to see how his ideas were received and how other observations were conducted at the time. Thus my research question is: Could Aristotle and Aristarchus describe the motion of the celestial bodies and the earth in a geocentric and heliocentric universe respectively? I will begin by comparing the reasoning of the two models; then I will asses if they could describe the motion of the heavenly bodies.

[1] Greek Astronomy, Thomas Heath, introduction liii

Greek Science:

I find it crucial to briefly study Greek Astronomy to answer the research question. There are two main influences on Greek science, Plato and Aristotle. Plato was influenced by the Pythagoreans in his ideas. They believed that the nature of things could be explained by geometry; and they believed in the migration of the soul. Plato formed the idea that the soul carried all experience from the past and thus contained all knowledge; you needed math and logic to get it out. Plato did therefore not believe it was necessary to observe in order to get knowledge. Plato ordered his earnest students to find "the uniform and ordered movement"[1] of the planets. Eudoxus found this movement by using three concentric spheres. This way of describing the way of the motion of the planets by eternal mathematical formulas is purely geometric and not mechanic, however to the platonians there was no difference in mechanics and geometry.

Aristotle also believed that knowledge can be derived from memory but that observations are what forms the memory and not memories in the soul carried forward from past lives. The Astronomers I have studied are Aristotelians. However Plato's influence over the Greek philosophers and astronomers was still immense even though their approach to science was different (they did more experiments) they still believed that everything could be described as geometrical figures. For the Greek a circular motion is a uniform motion. Something can move in a circle forever. This highlights their lack of understanding of inertia and acceleration.

Eratosthenes:

Eratosthenes knew that in Scyene there was a very deep well in which at the summer solstice the water at the bottom of the well reflected the sun light. He measured the suns angle of elevation in Alexandria, 5000 stadia away. The difference in the suns angle of elevation between Alexandria and Scyene was $7{,}2^{o}$ and hence the distance between it was one fiftieth of the earth's circumference. Since this was about 5000 stadia he calculated the earths circumference to be 250,000 stadia (he later revised this to 252,00 stadia).[2] The problem with this is that a stadia was a local measurement at the time, so the value varied from city to city. People writing about Eratosthenes generally like to point out how "good" his value is compared to todays measurement if one used "the best" value for a stadia, I find these discussions irrelevant. He managed to measure the angle in Alexandria very well, he missed that Scyene is not quite on the tropic of the cancer and Alexandria is not directly north. It is better to point out that for the time these values were excellent and they were much used by Aristarchus, Archimedes and later Alexandrian astronomers. The values in them selves are not interesting today.

Something which is interesting about Eratosthenes is that he only measured the earths curvature in north south direction. Technically that means he can only prove that the earth is a cylinder. To Eratosthenes this is not an issue. To him it is obvious that if the earth is not flat it is of course a sphere, because the sphere is the perfect shape. You could reason that to propose a flat earth would be less controversial than a spherical, but it would probably be the opposite. Maybe not be the case among commoners but the post-Pythagorean philosophers would find that theory disturbing.

I decided to try to conduct a similar experiment to Eratosthenes to "put" myself in his situation and see how one can easily conduct very simple experiments and get good values. I thought it would help me to understand how it was to conduct experiment in ancient Greece. I'm however limited in conducting the experiment. I don't have the time to travel nor do I

[1] Greek Astronomy, sir Thomas L. Heath, introduction xliv
[2] Greek Astronomy, sir Thomas L. Heath, page 109-112

have the conditions he had. The fact that his two measuring points are are almost directly north south and he has good values for the distance since the Nile connects both points.

I will therefore conduct two experiments to see how accurately you can measure the suns elevation. An advantage I have over the Greek scientists is an accurate sine table. I conducted the experiment in Scalea, Italy (39°47'N, 15°48'E), Skåne, Sweden (56°15'N 12°33'E)[1].

How to determine the earth is round:

The earth appears to be flat. If we go outside we cannot see the curvature of the earth. The easiest way to determine that the earth is spherical is by assuming the moon's light is a reflection of the sunlight; that the phases of the moon is the earth's shadow. By looking at this shadow, you see it's circular.[2] To prove this and to calculate the circumference you need to travel. Without the use of clocks you need to measure the difference in the suns angle of elevation at solar-noon and the distance between two points in north-south direction. From this the circular shape of the earth north-south is proven and the circumference calculated.

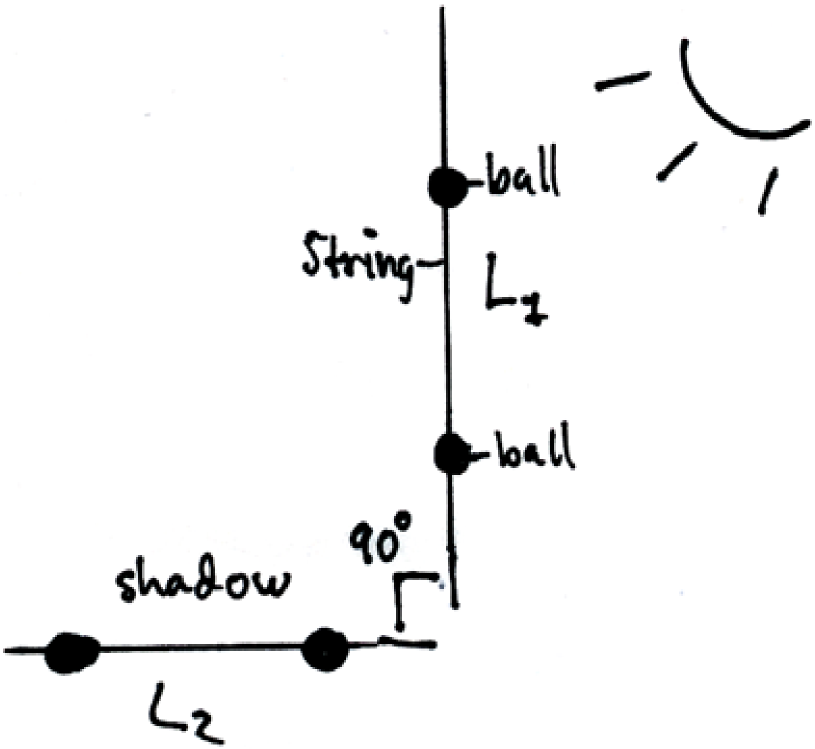
Experiment designated to determine angle of elevation:

Aim: To find the elevation of the sun at solar-noon at two different places to determine whether it is possible to determine the shape and circumference of the earth and this value.

Method: Two equally shaped balls were attached on a string with a weight on the end, hanging perpendicular to the earth. This casts a shadow which is measured at *n* minutes after t_0.

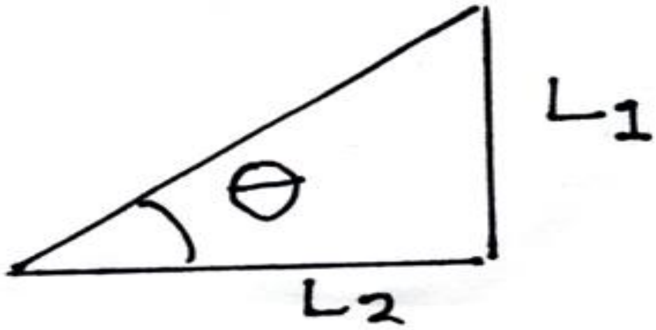
[1] Google Earth, satellite program.
[2] Greek Astronomy, Thomas Heath, page 1

Diagram 1. The apparatus of the angle of elevation experiment



The procedure is repeated over a time period which is somewhere around noon CET, to make sure the solar noon is included. The data is plotted with the length of the shadow L_2 against n minutes after t_0. A quadratic regression is performed and the y-coordinate of the vertex is the calculated angle of elevation. Assuming that the shadow is parallel to the ground and perpendicular to the string, the angle of elevation can be calculated using simple trigonometry.

Diagram 2. Angle of elevation. The length between the balls L_1 and its shadow L_2 and their relation to θ.



To calculate the angle of elevation, θ, I use the formula:

$$\theta = \arctan\left(\frac{L_1}{L_2}\right)$$

The difference in the angle of elevation represent a segment of a circle ϕ. Hence, by knowing the distance between the points, the earths circumference can be calculated.

Assuming ϕ is an arc of a longitude (great circle) μ

$$\text{Arc length } \mu_s = \text{ Arc length } \phi_s \times \frac{\phi}{360}$$

Since μ is a great circle of the earth, μ_s is the circumference.

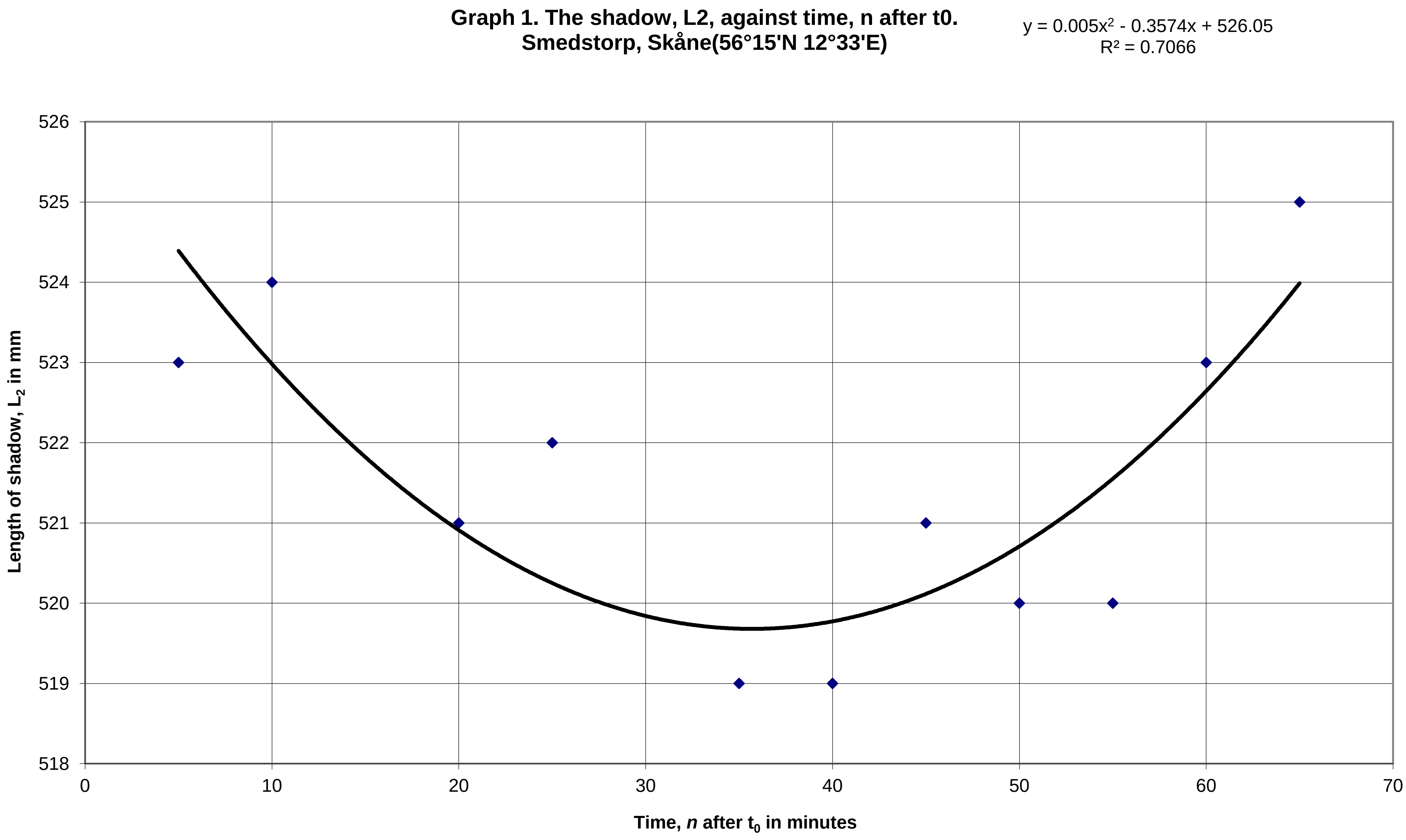
Smedstorp:

The distance between the balls: 755 mm
T_0 = 11.45 UniversalTime(UT)

Table of data:

Table 1. The length of the shadow L_2 around solar-noon on July 7th Smedstorp Skåne	
Time after t_0 (minutes)	Length of shadow (mm) ± 11mm
5	523
10	524
15	518
20	521
25	522
30	526
35	519
40	519
45	521
50	520
55	520
60	523
65	525

The uncertainty in the time is negligible, 0,0083 minutes.
The uncertainty for the string is also negligible, 0,5 mm.
I have not included these in any calculations because the are to small.
The uncertainty of the shadow is greater. The wind caused the string to swing about a centimetre back and forth. The uncertainty of the shadow is ± 1 mm + ±10 mm = ±11 mm



Graph 1. The shadow, L2, against time, n after t0.
Smedstorp, Skåne(56°15'N 12°33'E)
y = 0.005x² - 0.3574x + 526.05
R² = 0.7066
Length of shadow, L2 in mm
Time, n after t0 in minutes
526
525
524
523
522
521
520
519
518
0
10
20
30
40
50
60
70

Vertex of $0.005x^2 - 0.3574x + 526.05 =$

Axis of symetry $= \frac{-b}{2a} = \frac{0.3574}{0.01} = 35.74$

x coordinate of vertex = 35.74

y coordinate $= 0.005(35.74)^2 - 0.3574(35.74) + 526.05 = 519.66 \approx 520mm$

So the calculated value of the shadow is $520mm \pm 11mm$ at UT 12:21 (11:45+0:36)

$$\theta = \arctan\left(\frac{L_1}{L_2}\right) = \arctan\left(\frac{755}{520}\right) = 55.44^o \pm 1.1^o$$

To compare with the coordinate, $(90^o\text{-lat}) +$ Elevation of Sun to the equator July 7th UT 12

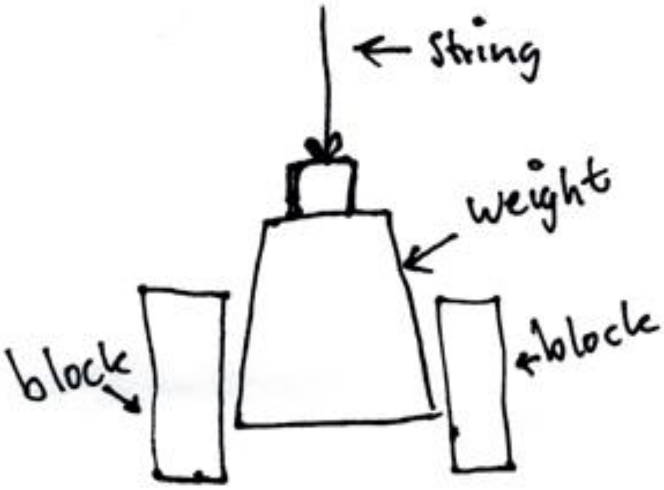
$= (90^o - 55.44^o = 34.56^o) + 22.57^o = 57.13^o \pm 1.1^o$

θ deviates 0.88^o from the textbook value, calculated using the calendar and the coordinates.

But this is within the error margin of the experiment and can therefore be considered a valid value.
[1]

To improve on this value, I chose to take more readings next time so that the r-squared value of the shadow-time graph is better and thus making the reading more accurate.
After this experiment I realised a big problem with the setup, keeping the string still and perpendicular to the ground due to the wind etc. In my next experiment this was solved by putting blocks around the weight at the bottom preventing the string to swing due to the wind.

Digram 3. The use of blocks to improve uncertainty.

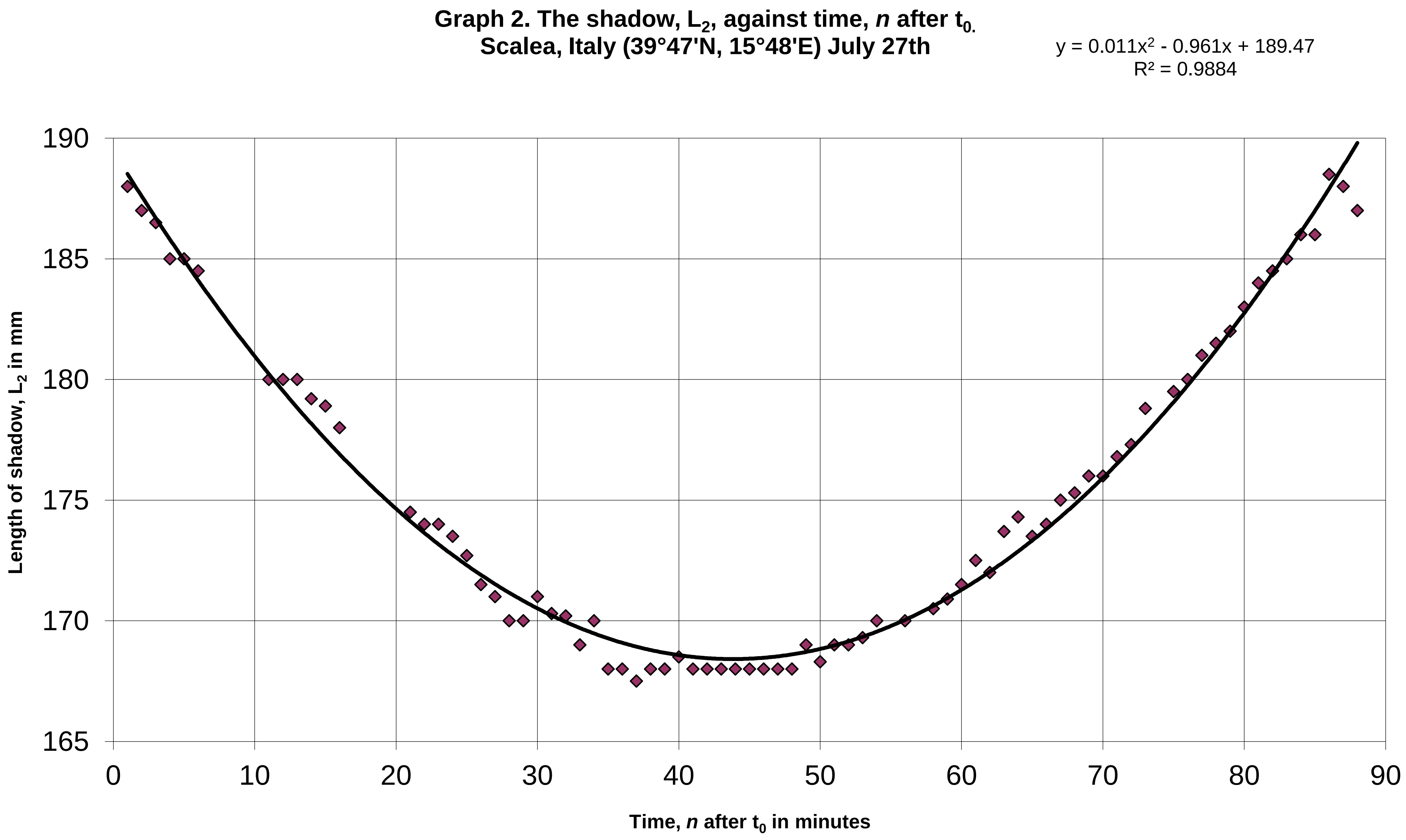


[1] Latitude from Nautical Almanac page 135

Scalea:
The distance between the balls: 450 mm
T_0 = 11.15
Sample table full table in appendix

Table 2. The length of the shadow L_2 around solar-noon on July 27th Scalea, Italy	
Time ***n*** after t_0 (**minutes**)	Length of shadow $\mathbf{L_2}$ (**mm**) ± 1mm
1	188
2	187
3	186,5
4	185
…	…
44	168
…	…
87	188
88	187

Here the uncertainty of the minutes is negligible because of the large range of data and the small uncertainty of 0.0083 minutes.
The shadow has an uncertainty of 1 mm.



Graph 2. The shadow, L2, against time, n after t0.
Scalea, Italy (39°47'N, 15°48'E) July 27th
y = 0.011x2 - 0.961x + 189.47
R² = 0.9884
Length of shadow, L2 in mm
190
185
180
175
170
165
0
10
20
30
40
50
60
70
80
90
Time, n after t0 in minutes

Vertex of $0.011x^2 - 0.961x + 198.5 =$

Axis of symetry $= \frac{\text{-b}}{\text{2a}} = \frac{0.961}{0.022} = 43.6818$

x coordinate of vertex = 43.6818

y coordinate $= 0.011(43.6818)^2 - 0.961(43.6818) + 198.5 = 168.48 \approx 168.5mm$

So the calculated value of the shadow is $168.5mm \pm 1mm$ at UT = 11:59 (11:15+00:44)

$$\theta = \arctan\left(\frac{L_1}{L_2}\right) = \arctan\left(\frac{450}{168.5}\right) = 69.47^o \pm 0.22^o$$

To compare with the coordinate, $(90^o\text{-lat}) +$ Elevation of Sun to the equator July 27th UT 12

$= (90^o - 69.47^o = 20.53^o) + 19.18^o = 39.71^o$

θ deviates 0.077^o from the textbook value, calculated using the calendar and the coordinates.

This within the uncertainty of the experiment which verifies the result.

This value is more accurate than the one previous one.

[1]

Given these two uncertainties; it is possible to measure the elevation of the sun using this method to ± 0.3 degrees. A limitation is that this method is not applicable to extreme values close to zero and ninety degrees. Another limitation is that I use far more developed math in my calculations than the Greek. They had nowhere near the accuracy of my calculator in their sine tables and they cannot use quadratic regression like I. However they could have advantages that I did not have. They could have arranged larger and more sophisticated apparatuses reducing the error in my equipment. And most importantly they could have repeated the experiment much more than I could. It is hard to conclude whether I or the Greeks could get more accurate results, but this experiment shows how simple it is to determine the shape of the earth and its perimeter. The perimeter can be calculated if you have a value between these points. I could not get any serious value between my locations without using unfair advantage like using GPS etc. which is a limitation to my experiment.

[1] [1] Latitude from Nautical Almanac page 149

Aristotle:

Aristotle believed the earth was spherical and at the centre of the universe. He had good reasons for this. He observed that objects which have weight fall down towards the centre of earth, explaining the spherical shape "for every one of its part has weight right down to the centre"[1]. To him it was clear that objects with mass were attracted to the earths centre, but why? His explanation was that the centre of the earth was at the centre of the universe. Not that the earth was the centre, merely that it was located at the centre. Fire moves (as observed on the earth) upward, therefore the sun, the planets and the stars, all made of fire, are in the opposite place, the edge. This theory explains gravity on earth and its spherical shape. What was the reason for things to be attracted to the earth if it was not situated at the middle of the universe, the place to which heavy objects are attracted.

Aristarchus of Samos:

Aristarchus wrote two works, one in which he measures and calculated the distance and size of the sun and the moon, in the other work he proposes that the sun and the fixed stars remain unmoved and that instead it is the earth which orbits the sun. This work is lost and is only known through citations in other works, like in *The Sand Reckoner* by Archimedes.

The Size and distance of the sun and the moon

Aristarchus writes these hypotheses in his first work[2]:

1. The moon receives its light from the sun
2. The moon moves in a sphere which the earth is the centre of
3. When the moon appears halved. the great circle which divides the moon into a dark and a light part is in the direction of the eye.
4. When the moon appears halved, the angle between the sun and the moon measured on the earth is 87^o.
5. The breadth of the earth shadow is two moon diameters.
6. The angular diameter of the moon is 2^o.

and given these hypotheses he concludes:

1. The distance between the sun and the earth, l, is $18 < l < 20$ times the distance between the earth and the moon.
2. The diameters of the sun and the moon have the same ratio as their respective distance to the earth.
3. The diameter of the sun compared to the diameter of the earth, d, is $\frac{19}{3} < d < \frac{43}{6}$.

[1] Greek Astronomy, Thomas Heath, page 91
[2] Greek Astronomy, sir Thomas L. Heath, page 100-101

Heliocentric view evolves

These conclusions, stating that the sun is about six to seven times larger than the earth, probably lead him to conclude what he wrote in his second work in which he proposes a hypothesis that the earth orbits the much larger sun. Archimedes writes about this work in *The Sand Reckoner*, where he describes Aristarchus universe, its and the stellar bodies sizes. Obviously Archimedes has read this book. Archimedes also claims that Aristarchus has discovered the angular diameter of the sun to be 0.5^o which suggest that after his first book Aristarchus revised his earlier value of 2^o. He likely used this new value for the sun and the moon in his second work since he must still have maintained that the sun and moon have the same angular diameter. This new angular diameter makes the sun and the moon four times further away than before however their real diameter remains unchanged and so they have the same size as in the earlier calculations.

Aristarchus heliocentric hypothesis

Archimedes describes Aristarchus hypothesis in the sand reckoner: "the fixed stars and the sun remain motionless; the earth has a circular orbit and the fixed stars are at the edge of a sphere, both which centre is the sun. The relationship between the diameter of the earth and the diameter of its circular orbit, is the same as, the diameter of the earth orbit and the diameter of the fixed star sphere."[1]

The Sand Reckoner

Archimedes invented a way of making very large numbers by using a sort of exponential system. The largest number of the time was a myriad myriad (10,000 * 10,000 or 100,000,000), Archimedes calculated the largest number possible without inventing a larger number than a myriad myriad as the base. The number was a myriad myriad to the myriad myraidth power all taken to the myriad myraidth power again, a one followed by eighty quadrillion zeros (reference, a googol is a one followed by a hundred zeros). The Sand Reckoner is written to use this new number and put it into context. At the time, many people thought that the number of sand grains was too large to count. Therefore Archimedes decides to count the number of sand grains in the universe filled with sand and show that even this number is ridiculously small compared to his number. Archimedes clearly exaggerates all numerical data in order for his number to appear even more fantastic. This work is important to this essay because Archimedes puts Aristarchus new universe into real values and it is also the primary source of Aristarchus heliocentric model. The reason for using Aristarchus system is that it is much larger than the older geocentric system; the size of the heliocentric universe in earth radii is essentially the geocentric universe squared. In order for a heliocentric system too work, the fixed stars must be much further away due to the apparent lack of parallax. Archimedes also makes the earth 10 times larger and the sun 3/2 times larger; which makes the distance between the earth and the sun, the earth and the stars, 15 and 27,5 respectively, times larger than previously believed. This gives a universe which has the size of the old universe squared and times 27,5 which is clearly a lot larger. Then Archimedes calculates the grain of sand in this universe and found it to be 10^{64}, which is nowhere near the number he invented. This shows the problem with a much larger universe to cope with, it will be explained more in the stellar parallax section.

[1] Greek Astronomy, Thomas Heath, page 106

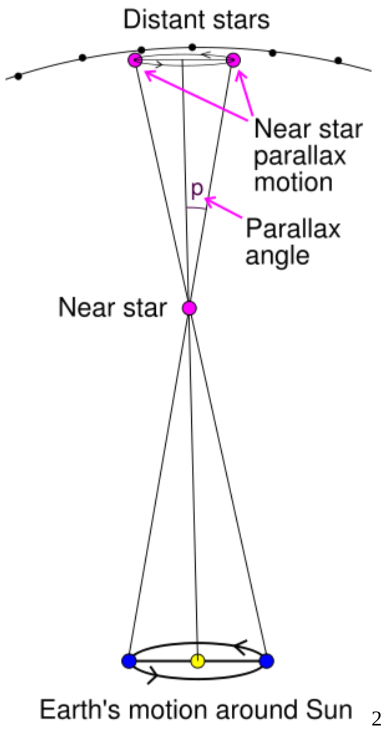
Problems with Aristarchus model:

There are two consequences of the heliocentric model: the rotating earth and the extreme distance to the stars. Aristotle addresses both:

The arrow problem:

Jean Buridan, a French medieval philosopher describes Aristotle's counter-argument to the heliocentric model: "an arrow projected from a bow directly upward falls to the same spot on the earth from which it was projected. This would not be so if the earth were moved with such velocity. Rather, before the arrow falls, the part of the earth from which the arrow was projected would be a league's distance away."[1] Showing the lack of understanding of inertia. This is actually true to some extent, since the arrow is shot up, it has to travel an arc which is larger than that of the observer on the earth. With an arrow this effect is not noticeable. The point is however, that Aristotle has not realised that the arrow has an inertial movement compared to the ground and will therefore follow the ground, even though not in direct contact to it. This is the most important counter-argument because it puts the finger on a very important misunderstanding of physics.

Diagram 4. Stellar parallax



[2]

note: parallax is the motion of a star relative to a fixed point, not necessarily another star

Stellar parallax:

Aristotle says "If [orbit of the earth] occurred, it would follow that the stars would exhibit passings and turningsback. This does not, however, to be the case, but the same stars always rise and set at the same places on the earth".[3] It is interesting that he doesn't rule it out, just simply explains that it is not visible (which is in fact not very strange as the largest stellar parallax is as large as a post stamp appears on a distance of 5.3 kilometres).

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[1] http://www.clas.ufl.edu/users/rhatch/HIS-SCI-STUDY-GUIDE/0039_jeanBuridan.html 2006-11-17

[2] Image from http://en.wikipedia.org/wiki/Image:Stellarparallax2.svg, free from copyright, released to public domain

[3] Greek Astronomy, sir Thomas L. Heath, page 89

This is a table which shows the distance to the fixed stars from the sun with the given parallax. The last value is the stellar parallax of the closest star, Proxima Centauri. This table shows the enormous expansion of the universe you get if you accept the heliocentric model.

Table 3. The effect of the angular stellar parallax on the universe		
Angle of parallax	Distance to the stars in earth orbits	Angular radius of the sun at this distance
90	1	0,5
45	2,414213562	0,2071079
10	11,4300523	0,0437446
1	114,5886501	0,0043635
10'	190,9841864	0,002618
1'	6875,493493	0,0000727
1"	412529,6125	0,0000012
0,77"	535752,7435	0,0000009

Let us consider a measurable parallax value at the time of 1,5 degrees. That would make the universe some dozens larger than previously anticipated but nothing extremely drastic. Let us now consider Jupiter which is one of the brighter celestial objects on the sky and has an angular diameter of 0,0065 degrees which is roughly measurable (about half an arc minute). To the Greeks planets were merely moving stars. Therefore you could say Alpha Centauri, having the same apparent brightness magnitude, has the same angular diameter as Jupiter. [This was later discovered to be an optical illusion by Galileo; the stars have smaller diameters than perceived by the eye.]
This would mean that Alpha Centauri is as large as the sun. Now we have a major problem, the reason for putting the sun in the middle in the first place was that it was the largest. Now there are hundreds of other "suns" in the universe and everything becomes very large and the earth insignificant. Obviously this is a major problem with the Heliocentric-model.

Analysis:

The research question was: Is it possible to describe the orbits of the planets, the earth, the sun and the moon, with the physics of Aristarchus, Archimedes and Ptolemy? The key word is **describe**. To describe in physics does not mean describe like you describe a painting. It means to describe through physical law. The motion of an object can be described with vectors and mechanics. It is not to know the reason behind a phenomena, that is metaphysical. So, to answer the question; yes, with a geocentric model. It predicts the motions of the sun, stars and planets. The motion of the heavenly objects is more complicated to describe but it is very easy to understand and explain. There are no problems with the system because the concepts of inertia, acceleration and gravity are not understood. It actually works perfectly. The heliocentric system on the other hand is an easier model, but not possible to describe; the lack of inertia causing the arrow problem and the parallax problem. However as science and technology advances, the discovery that the planets are other earth like bodies and the understanding of inertia makes the geocentric model impossible to understand. Physics moves on and with the use of gravity the heliocentric solar-system replaces the geocentric model. However the heliocentric model is still not complete.

Applying the research question to a modern example; Relativity, gives the same answer. Is it possible to understand the speed of light without relativity? The answer is yes as long as it is assumed that the speed of light is not constant and with the concept of an "ether wind". Off course, at one point this old thinking has to give way to new because of discoveries, such as the Michelson-Morley experiment, making one of these obsolete. The way Newton's law of Gravity rendered geocentrism obsolete and the way Einstein in turn made Newton’s laws of mechanics a special case of the special and general theory of relativity. Both of these examples drastically change the old theories and physics takes a leap. This is an important part of physics!

Conclusion:

Well, could the Greek describe the motions of the stars, sun and planets in a geocentric and heliocentric respectively? The answer is yes and no. Yes they could describe a geocentric universe; but they could not describe a heliocentric. This is best shown by the counter argument with the arrow. The lack of understanding of inertia could not let the Greeks describe a heliocentric model. Their concept of inertia, that an object will eventually stop is that which works on earth. This can be applied to the relativity problem. On earth it seems that motion is relative to the observer but Einstein shows that this is not the case with light. In a similar way which Galileo shows that an object will not change velocity unless acted upon by a force, which is friction here on earth. From this we learn the process of physical thinking: in order to explain a new phenomena, old laws may have to be changed.

Bibliography:

Greek Astronomy, Sir Thomas L. Heath, New York: Dover Publications 1991
Grekisk vetenskap, Benjamin Farrington, Halmstad: Prisma 1965
http://en.wikipedia.org/wiki/Plato 2006-08-23
http://en.wikipedia.org/wiki/Aristotle 2006-08-23
http://en.wikipedia.org/wiki/Aristarchus_of_Samos 2006-08-23
http://en.wikipedia.org/wiki/Eratosthenes 2006-08-23
http://www.clas.ufl.edu/users/rhatch/HIS-SCI-STUDY-GUIDE/0039_jeanBuridan.html 2006-11-17
Nautical Almanac 2006 Commercial Edition, Celestaire Inc.m Wichita 2005.
http://en.wikipedia.org/wiki/Stellar_parallax 2006-11-17

Appendix: Data from Scalea.

Time after t_0 (minutes)	Length of shadow (mm) ± 1mm
1	188
2	187
3	186,5
4	185
5	185
6	184,5
7	
8	
9	
10	
11	180
12	180
13	180
14	179,2
15	178,9
16	178
17	
18	
19	
20	
21	174,5
22	174
23	174
24	173,5
25	172,7
26	171,5
27	171
28	170
29	170
30	171
31	170,3
32	170,2
33	169
34	170
35	168
36	168
37	167,5
38	168
39	168
40	168,5
41	168
42	168
43	168
44	168
45	168
46	168
47	168

48	168
49	169
50	168,3
51	169
52	169
53	169,3
54	170
55	
56	170
57	
58	170,5
59	170,9
60	171,5
61	172,5
62	172
63	173,7
64	174,3
65	173,5
66	174
67	175
68	175,3
69	176
70	176
71	176,8
72	177,3
73	178,8
74	
75	179,5
76	180
77	181
78	181,5
79	182
80	183
81	184
82	184,5
83	185
84	186
85	186
86	188,5
87	188
88	187